A photon rest mass and electric currents in the Galaxy

1973 J. Phys. A: Math. Nucl. Gen. 6 L12

(http://iopscience.iop.org/0301-0015/6/2/003)

J. Phys. A: Math., Nucl. Gen., Vol. 6, February 1973. Printed in Great Britain. © 1973

LETTER TO THE EDITOR

A photon rest mass and electric currents in the Galaxy

J C Byrne and R R Burman

Department of Physics, University of Western Australia, Nedlands, WA 6009, Australia

MS received 31 October 1972, in revised form 20 December 1972

Abstract. Following an idea of Goldhaber and Nieto, conditions for the existence and stability of the currents which are required to support the observed galactic magnetic field are used to obtain an upper limit for the photon rest mass.

The established upper limit on the photon rest mass m is that obtained by Goldhaber and Nieto (1968); they used a method introduced by Schrödinger (1943), which is based on measurements of the geomagnetic field, and showed that the reduced Compton wavelength μ^{-1} of the photon exceeds 10^{10} cm. Williams and Park (1971) introduced a method for obtaining a limit on m which is based on the dissipation of magnetic fields in the Galaxy; it has been claimed (Byrne and Burman 1972) that their treatment needs modification, after which the method shows that $\mu^{-1} \gtrsim 3\times10^{10}$ cm. In their review of theoretical and experimental work on the photon rest mass, Goldhaber and Nieto (1971) suggested another astrophysical method: since there is a maximum current density that a plasma can support, knowledge of magnetic fields in the Galaxy can be used to obtain a limit on m; they deduced, without giving detailed arguments, that $\mu^{-1} \gtrsim 10^{15}$ cm. This method will be developed here.

The effect of a nonzero photon rest mass can be incorporated into electrodynamics by replacing Maxwell's equations with their simplest relativistic generalization, the Proca equations; these lead to $\Box^2 \boldsymbol{A} - \mu^2 \boldsymbol{A} = -(4\pi/c)\boldsymbol{j}$ where $\boldsymbol{A}$ is the vector potential, which is related to the magnetic field $\boldsymbol{H}$ by $\boldsymbol{H} = \nabla \times \boldsymbol{A}$, c is the maxwellian speed of light in free space and $\boldsymbol{j}$ is the current density; A and H will denote $|\boldsymbol{A}|$ and $|\boldsymbol{H}|$. Take $|\Box^2 \boldsymbol{A}| \simeq AL_1{}^{-2}$, so that L_1 is a characteristic length over which A varies significantly. If $L_1{}^2 \gg \mu^{-2}$, then the above equation for A leads to

$$\mu^2 \lesssim \frac{4\pi}{c}\frac{j_{\mathrm{m}}}{A} \qquad (1)$$

where j_{m} denotes the magnitude of the maximum current density that could exist in the plasma. Take $A \simeq HL_2$, so that L_2 is another characteristic length over which A varies significantly. For magnetohydrodynamic conditions, both L_1 and L_2 will usually be the smallest dimension of a quasi-uniform magnetic field. The galactic magnetic field is observed to be ordered over distances of several hundred parsec and to have a strength of about 3×10^{-6} G (Mathewson and Nicholls 1968, Manchester 1972).

It has been shown (Jacobsen and Carlqvist 1964, Alfvén and Carlqvist 1967, Alfvén 1968, Carlqvist 1969) that a plasma becomes unstable and locally evacuated, with a corresponding large fall in conductivity, when the electron drift speed V exceeds the electron thermal speed U_e. Stix (1962 chap 9) discussed instabilities that occur when V exceeds the phase speeds of some wave types. The magnetosonic overstability arises when V exceeds the Alfvén speed V_A; the same overstability will occur, owing to excitation of the ion cyclotron wave, when $V \gtrsim (V_A{}^2 U_i)^{1/3}$ where V_A refers to the Alfvén wave in the electron–ion fluid and U_i is the ion thermal speed. The observed critical electron drift speed in laboratory plasmas is around U_i (Stix 1962 p 203, Alfvén 1968). If V_m denotes the maximum electron drift speed that the plasma can support stably, then $j_m = neV_m$ where n is the electron number density and e is the magnitude of the electronic charge. Since $V_A = H/(4\pi\rho)^{1/2}$ where ρ is the density of the medium, $j_m/A \simeq ne/L_2(4\pi\rho)^{1/2}$ when $V_m = V_A$: the right side of (1) is independent of H in this case, but is not independent of $\boldsymbol{H}$ since the geometry of $\boldsymbol{H}$ determines L_2.

In galactic HI regions, $n \lesssim 10^{-2}\,\mathrm{cm}^{-3}$ (Rees and Sciama 1969). In HI clouds, the temperature T is less than about 10^2 K (Kerr 1968 see § 7.1) so that $U_i \lesssim 10^5$ cm s^{-1}. The intercloud medium could have $T \sim 10^4$ K (Field 1969) so that $U_i \sim 10^6$ cm s^{-1}. Observations of the profiles of spectral lines show that $U_i \lesssim 7\times10^5\,\mathrm{cm\,s^{-1}}$ in interstellar clouds (van Woerden 1967). Let n_H denote the number density of hydrogen atoms. For HI regions, since n_H is between about $10^3\,\mathrm{cm}^{-3}$ and about $0{\cdot}3\,\mathrm{cm}^{-3}$ or perhaps less (Rees and Sciama 1969), $V_A/H \sim 10^{10}$ to $\sim 3\times10^{11}\,\mathrm{g^{-1/2}\,cm^{3/2}}$ or more (corresponding, with $H \sim 3\times10^{-6}$ G, to $V_A \sim 3\times10^4$ to $\sim 10^6\,\mathrm{cm\,s^{-1}}$ or more) for Alfvén waves in the electron–ion–neutral fluid of such regions. Also $nV_A/H \lesssim 3\times10^{10}$ $\mathrm{g^{-1/2}\,cm^{-3/2}}$ (corresponding, with $H \sim 3\times10^{-6}$ G, to $V_A \gtrsim 10^7\,\mathrm{cm\,s^{-1}}$) for Alfvén waves in the electron–ion fluid both in clouds and in the intercloud medium. Hence, V_m will be around $10^5\,\mathrm{cm\,s^{-1}}$ in cool HI regions and around $10^6\,\mathrm{cm\,s^{-1}}$ in a hot intercloud medium. Taking $H \sim 3\times10^{-6}$ G, $n \lesssim 10^{-2}\,\mathrm{cm}^{-3}$ and $L_2 \gtrsim 10^{20}$ cm, so that $A \gtrsim 3\times10^{14}$ G cm, gives $\mu \lesssim 3\times10^{-15}\,\mathrm{cm}^{-1}$ or $\mu \lesssim 10^{-15}\,\mathrm{cm}^{-1}$ according to whether the intercloud medium is hot or cold.

The various instabilities discussed above all lead to roughly similar limitations on the electron drift speed in the interstellar medium: it is clearly indicated that V_m will be around $10^5\,\mathrm{cm\,s^{-1}}$ in cool clouds and around $10^6\,\mathrm{cm\,s^{-1}}$ in a hot intercloud medium. The latter limit is determined by the electron–ion sound coupling (Alfvén 1968), which determines the former limit also, except in clouds of high density where the magnetosonic overstability, arising through excitation of the Alfvén wave in the electron–ion–neutral fluid, might reduce V_m to about $3\times10^4\,\mathrm{cm\,s^{-1}}$.

A maximum value for the current density in the interstellar medium can be obtained in another way: the rate Λ at which thermal energy is lost, by all processes, can be calculated (Spitzer 1968 § 4.4, Kaplan and Pikelner 1970 p 127) and, in thermal equilibrium, the rate of Joule heating j^2/σ, where σ denotes the conductivity, cannot exceed Λ; hence, $j_m \leqslant (\sigma\Lambda)^{1/2}$. Reference to the standard formula for σ (Byrne and Burman 1972) shows that $\sigma \lesssim 5\times10^9\,\mathrm{s}^{-1}$ for $T \lesssim 10^2$ K and $\sigma \sim 3\times10^{12}\,\mathrm{s}^{-1}$ for $T \sim 10^4$ K. According to Kaplan and Pikelner, $\Lambda \sim 6\times10^{-25}\,\mathrm{erg\,s^{-1}\,cm^{-3}}$ for $T \sim 10^2$ K, $n \sim 3\times10^{-2}\,\mathrm{cm}^{-3}$ and $n_H \sim 2\,\mathrm{cm}^{-3}$, while $\Lambda \sim 6\times10^{-24}\,\mathrm{erg\,s^{-1}\,cm^{-3}}$ for $T \sim 10^4$ K, $n \sim 10^{-2}\,\mathrm{cm}^{-3}$ and $n_H \sim 3\times10^{-2}\,\mathrm{cm}^{-3}$. Thus for $A \gtrsim 3\times10^{14}$ G cm, (1) shows that $\mu \lesssim 3\times10^{-15}\,\mathrm{cm}^{-1}$ or $\mu \lesssim 3\times10^{-16}\,\mathrm{cm}^{-1}$ according to whether the intercloud medium is hot or cold. According to Spitzer (1968 p 138), $\Lambda \sim 2\times10^{-26}\,\mathrm{erg\,s^{-1}\,cm^{-3}}$ for $T \sim 10^2$ K, $n \sim 10^{-2}\,\mathrm{cm}^{-3}$ and $n_H \sim 1\,\mathrm{cm}^{-3}$; with this value of Λ, (1) shows that $\mu \lesssim 10^{-16}\,\mathrm{cm}^{-1}$ if the intercloud medium is cold.

The stability considerations and the Joule dissipation argument provide similar upper limits on μ. The main source of uncertainty in the calculations occurs in the determination of L_2; this uncertainty will be reduced with improved knowledge of the geometry of the galactic magnetic field.

The calculations presented here, using the method suggested by Goldhaber and Nieto (1971), establish that $\mu \leqslant 3\times10^{-15}$ cm^{-1}, corresponding to $m \leqslant 10^{-52}$ g and to $\mu^{-1} \geqslant 3\times10^{14}$ cm or $\geqslant$ 30 AU.

References

Alfvén H 1968 *Ann. Geophys.* **24** 341–6

Alfvén H and Carlqvist P 1967 *Solar Phys.* **1** 220–8

Byrne J C and Burman R R 1972 *J. Phys.* A: *Gen. Phys.* **5** L109–11

Carlqvist P 1969 *Solar Phys.* **7** 377–92

Field G B 1969 *Comm. Astrophys. space Phys.* **1** 107–15

Goldhaber A S and Nieto M M 1968 *Phys. Rev. Lett.* **21** 567–9

—— 1971 *Rev. mod. Phys.* **43** 277–96

Jacobsen C and Carlqvist P 1964 *Icarus* **3** 270–2

Kaplan S A and Pikelner S B 1970 *The Interstellar Medium* (Cambridge, Massachusetts: Harvard University Press)

Kerr F J 1968 *Nebulae and Interstellar Matter* ed B M Middlehurst and L H Aller (Chicago and London: University of Chicago Press) pp 575–622

Manchester R N 1972 *Astrophys. J.* **172** 43–52

Mathewson D S and Nicholls D C 1968 *Astrophys. J.* **154** L11–6

Rees M J and Sciama D W 1969 *Comm. Astrophys. space Phys.* **1** 35–40

Schrödinger E 1943 *Proc. R. Irish Acad.* A **49** 135–48

Spitzer L 1968 *Diffuse Matter in Space* (New York: Interscience)

Stix T H 1962 *The Theory of Plasma Waves* (New York: McGraw-Hill)

van Woerden H 1967 *Radio Astronomy and the Galactic System* ed H van Woerden (London and New York: Academic Press) pp 3–28

Williams E and Park D 1971 *Phys. Rev. Lett.* **26** 1651–2